P 53

[1] $\cos 75 = \cos(45+30)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

$\cos 15 = \cos(45-30)$

$= \cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

[2] $\sin 105^\circ = \sin(60+45)$

$= \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

$\longrightarrow$

[2] ctd

$$\cos 105 = \cos(60+45)$$
$$= \cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$$
$$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$
$$= \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$
$$= \frac{\sqrt{2}-\sqrt{6}}{4}$$

P54

[3] $\sin\alpha = \frac{4}{5}$, $\cos\beta = -\frac{8}{17}$, $0 < \alpha < \frac{\pi}{2}$, $\frac{\pi}{2} < \beta < \pi$,

$$\cos^2\alpha = 1-\sin^2\alpha = 1-\frac{16}{25} = \frac{9}{25} \Rightarrow \boxed{\cos\alpha = \frac{3}{5}}$$
$$\sin^2\beta = 1-\cos^2\beta = 1-\frac{64}{289} = \frac{225}{289} \Rightarrow \boxed{\sin\beta = \frac{15}{17}}$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \sin\beta\cos\alpha$$
$$= \frac{4}{5}\cdot\frac{-8}{17} - \frac{15}{17}\cdot\frac{3}{5}$$
$$= \frac{1}{17\cdot 5}\left[-32-45\right]$$
$$= \frac{-77}{17\cdot 5}$$
$$= \frac{-77}{85}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \frac{3}{5}\cdot\frac{-8}{17} - \frac{4}{5}\cdot\frac{15}{17}$$
$$= \frac{1}{85}\left[-24-45\right]$$
$$= -\frac{69}{85}$$

$\longrightarrow$

P54 [3] ctd

$$\cos(\alpha-\beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$$
$$= \frac{3}{5}\cdot\frac{15}{17} + \frac{4}{5}\cdot\frac{15}{17}$$
$$= \frac{15+45}{85}$$
$$= \frac{60}{85}$$
$$= \frac{12}{17}$$

[4]

$$\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta}$$
$$= \frac{\sin\alpha\cos\beta + \sin\beta\cos\alpha}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} \cdot \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}}$$
$$= \frac{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}}{\frac{\cos\beta}{\cos\beta} - \frac{\sin\alpha}{\cos\alpha}\frac{\sin\beta}{\cos\beta}}$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

[5]

$$\tan 15° = \tan(45° - 30°)$$
$$= \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$$
$$= \frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$
$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1\cdot\frac{\sqrt{3}}{3}}$$
$$= \frac{1 - \sqrt{3}/3}{1 + \sqrt{3}/3}$$

$\longrightarrow$

P 54 [5] ctd

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} \cdot \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{12-6\sqrt{3}}{6}$$

$$= 2 - \sqrt{3}$$

$$\tan 75 = \operatorname{Tan}(45° + 30°)$$
$$= \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$
$$= \frac{\tan\frac{\pi}{4} + \tan\frac{\pi}{6}}{1 - \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$$
$$= \frac{1 + \frac{\sqrt{3}}{3}}{1 - 1 \cdot \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{3-\sqrt{3}} = \frac{12+6\sqrt{3}}{6}$$
$$= 2 + \sqrt{3}$$

$$\tan(105°) = \tan(60° + 45°) = \tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$
$$= \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$$
$$= \frac{\sqrt{3} + 1}{1 - \sqrt{3}(1)} = \frac{1+\sqrt{3}}{1-\sqrt{3}}$$
$$= \frac{4+2\sqrt{3}}{-2}$$
$$= -(2 + \sqrt{3})$$

P55

[1]

$$\cos 2\alpha = \cos(\alpha+\alpha)$$
$$= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$$
$$= \cos^2\alpha - \sin^2\alpha$$

$$\tan 2\alpha = \tan(\alpha+\alpha)$$
$$= \frac{\tan\alpha+\tan\alpha}{1-\tan\alpha\tan\alpha}$$
$$= \frac{2\tan\alpha}{1-\tan^2\alpha}$$

[2] $(\sin\alpha + \cos\alpha)^2 = 1+\sin 2\alpha$

$$LHS = (\sin\alpha+\cos\alpha)^2$$
$$= \sin^2\alpha + \cos^2\alpha + 2\sin\alpha\cos\alpha$$
$$= 1 + 2\sin\alpha\cos\alpha$$
$$= 1+\sin 2\alpha$$

$\square$

$$\cos^4\alpha - \sin^4\alpha = \cos 2\alpha$$
$$LHS = \cos^4\alpha - \sin^4\alpha$$
$$= (\cos^2\alpha - \sin^2\alpha)(\cos^2\alpha + \sin^2\alpha)$$
$$= \cos 2\alpha$$

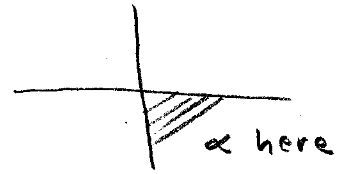
$\square$

$\longrightarrow$

[3] If $\sin\alpha = -\frac{1}{2}$ and $-\frac{\pi}{2} < \alpha < 0$

Then $\cos^2\alpha = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow \boxed{\cos\alpha = \frac{\sqrt{3}}{2}}$



$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$
$$= 2\left(-\frac{1}{2}\right)\frac{\sqrt{3}}{2}$$
$$= -\frac{\sqrt{3}}{2}$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$
$$= \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$\therefore \cos 2\alpha = \frac{1}{2}$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

$$= \frac{2\left(-\frac{1}{\sqrt{3}}\right)}{1-\frac{1}{9}}$$
$$= -\frac{\frac{2}{\sqrt{3}}}{\frac{8}{9}}$$
$$= -\frac{2}{\sqrt{3}}\cdot\frac{9}{8}$$
$$= -\frac{1}{4}\ \frac{9}{\sqrt{3}}$$
$$= -\frac{9\sqrt{3}}{3}$$
$$= -3\sqrt{3}$$

p 56 ctd

[4.1] Prove $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$$
\begin{aligned}
LHS &= \sin 3\alpha \\
&= \sin 2\alpha \cos\alpha + \sin\alpha \cos 2\alpha \\
&= 2\sin\alpha \cos^2\alpha + \sin\alpha\,(1 - 2\sin^2\alpha) \\
&= 2\sin\alpha \cos^2\alpha + \sin\alpha - 2\sin^3\alpha \\
&= 2\sin\alpha\,(1 - \sin^2\alpha) + \sin\alpha - 2\sin^3\alpha \\
&= 2\sin\alpha - 2\sin^3\alpha + \sin\alpha - 2\sin^3\alpha \\
&= 3\sin\alpha - 4\sin^3\alpha
\end{aligned}
$$

$\square$

[4.2] Prove $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$

$$
\begin{aligned}
LHS &= \cos 3\alpha \\
&= \cos 2\alpha \cos\alpha - \sin 2\alpha \sin\alpha \\
&= \cos 2\alpha \cos\alpha - 2\sin^2 \cos\alpha \\
&= [2\cos^2\alpha - 1]\cos\alpha - 2(1 - \cos^2\alpha)\cos\alpha \\
&= 2\cos^3\alpha - \cos\alpha - 2\cos\alpha + 2\cos^3\alpha \\
&= 4\cos^3\alpha - 3\cos\alpha
\end{aligned}
$$

$\square$

$\rightarrow$

P56 ctd

[5]

Prove $\sin^2 \frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$

Proof

$$\cos\alpha = 1 - 2\sin^2\frac{\alpha}{2}$$ [sub α for 2α into $\cos 2\alpha = 1 - 2\sin^2\alpha$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

□

Prove $\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$

Proof

$$\cos\alpha = 2\cos^2\frac{\alpha}{2} - 1$$ [sub α for 2α into $\cos 2\alpha$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

□

Prove $\tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$

Proof

$$\tan^2\frac{\alpha}{2} = \frac{\sin^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}$$

$$= \frac{\frac{1-\cos\alpha}{2}}{\frac{1+\cos\alpha}{2}}$$

$$= \frac{1-\cos\alpha}{1+\cos\alpha}$$

* where $\alpha \neq \pi + 2n\pi,\ n \in \mathbb{Z}$

I.E. Domain $= \mathbb{R} - \{\alpha : \alpha = \pi + 2n\pi,\ n \in \mathbb{Z}\}$